VI

Deductive Arguments

Consider this argument:

> If there are no chance factors in chess, then chess is a game of pure skill.
>
> There are no chance factors in chess.
>
> Therefore, chess is a game of pure skill.

Suppose that the premises of this argument are true. In other words, suppose it's true that *if* there are no chance factors in chess, then chess is a game of pure skill—and suppose there *are* no chance factors in chess. You can therefore conclude with perfect assurance that chess is a game of pure skill. There is no way to admit the truth of these premises but deny the conclusion.

Arguments of this type are called *deductive arguments*. That is, a (properly formed) deductive argument is an argument of such a form that if its premises are true, the conclusion must be true too. Properly formed deductive arguments are called *valid* arguments.

Deductive arguments differ from the sorts of arguments so far considered, in which even a large number of true premises does not guarantee the truth of the conclusion (although sometimes they may make it very likely). In nondeductive arguments, the conclusion unavoidably goes beyond the premises—that's the very point of arguing by example, authority, and so on—whereas the conclusion of a valid deductive argument only makes explicit what is already contained in the premises.

In real life, of course, we can't always be sure of our premises either, so the conclusions of real-life deductive arguments still have to be taken with a few (sometimes many) grains of salt. Still, when strong premises can be found, deductive forms are very useful. And even when the premises are uncertain, deductive forms offer an effective way to

organize arguments.

22 *Modus ponens*

Using the letters **p** and **q** to stand for declarative sentences, the simplest valid deductive form is

If [sentence **p**] then [sentence **q**].

[Sentence **p**].

Therefore, [sentence **q**].

Or, more briefly:

If **p** then **q.**

p.

Therefore, **q.**

This form is called *modus ponens* ("the mode of putting": put **p, get q**). Taking **p** to stand for "There are no chance factors in chess," and **q** to stand for "Chess is a game of pure skill," our

introductory example follows *modus ponens* (check it out). Here is another:

> If drivers on cell phones have more accidents, then drivers should be prohibited from using them.
>
> Drivers on cell phones *do* have more accidents.
>
> Therefore, drivers should be prohibited from using cell phones.

To develop this argument, you must explain and defend both of its premises, and they require quite different arguments (go back and look). *Modus ponens* gives you a way to lay them out clearly and separately from the start.

23 *Modus tollens*

A second valid deductive form is *modus tollens* ("the mode of taking": take **q**, take **p**).

> If **p** then **q**.

Not-**q.**

Therefore, not-**p.**

Here "Not-**q**" simply stands for the denial of **q,** that is, for the sentence "It is not true that **q.**" The same is true for "not-**p.**"

Remember Sherlock Holmes's argument, discussed under Rule 1:

> A dog was kept in the stables, and yet, though someone had been in and had fetched out a horse, [the dog] had not barked. ... Obviously the ... visitor was someone whom the dog knew well.

Holmes's argument can be put as a *modus tollens:*

If the visitor were a stranger, then the dog would have barked.

The dog did not bark.

Therefore, the visitor was not a stranger.

To write this argument in symbols, you could use **s** for "The visitor was a stranger" and **b** for "The dog barked."

If **s** then **b.**

Not-**b.**

Therefore, not-**s.**

"Not-**b**" stands for "The dog did not bark," and "not-**s**" stands for "The visitor was not a stranger." As Holmes puts it, the visitor was someone whom the dog knew well.

24 Hypothetical syllogism

A third valid deductive form is "hypothetical syllogism."

If **p** then **q.**

If **q** then **r.**

Therefore, if **p** then **r.**

For instance:

> If you **s**tudy other cultures, then you start to realize the **v**ariety of human customs.
>
> If you start to realize the **v**ariety of human customs, then you become more **t**olerant.
>
> Therefore, if you **s**tudy other cultures, then you become more **t**olerant.

Using the letters in boldface to stand for the component sentences in this statement, we have:

> If **s** then **v.**
>
> If **v** then **t.**
>
> Therefore, if **s** then **t.**

Hypothetical syllogisms are valid for any number of premises, as long as each premise has the form "If **p** then **q**" and the **q** (called the "consequent") of one premise becomes the **p** (the "antecedent") of the next.

25 Disjunctive syllogism

A fourth valid deductive form is “disjunctive syllogism.”

> **p** or **q**.
>
> Not-**p**.
>
> Therefore, **q**.

Consider, for instance, Bertrand Russell’s argument discussed under Rule 2:

> Either we hope for progress by improving **m**orals or we hope for progress by improving **i**ntelligence.
>
> We can’t hope for progress by improving **m**orals.
>
> Therefore, we must hope for progress by improving **i**ntelligence.

Again using the boldface letters as symbols,

this argument goes

m or **i.**

Not-**m.**

Therefore, **i.**

There is one complication. In English the word “or” can have two different meanings. Usually “**p** or **q**” means that at least one of **p** or **q** is true, and possibly both. This is called an “inclusive” sense of the word “or” and is the sense normally assumed in logic. Sometimes, though, we use “or” in an “exclusive” sense, in which “**p** or **q**” means that either **p** or **q** is true but *not* both. “Either they’ll come by land or they’ll come by sea,” for example, suggests that they won’t come both ways at once. In that case you might be able to infer that if they come one way, then they’re *not* coming the other way (better be sure!).

Disjunctive syllogisms are valid regardless of which sense of “or” is used (check it out). But what *else*, if anything, you may be able to infer from a statement like “**p** or **q**”—in

particular, whether you can conclude not-**q** if you also know **p**—depends on the meaning of "or" in the specific "**p** or **q**" premise you are considering. Take care!

26 Dilemma

A fifth valid deductive form is the "dilemma."

p or **q.**

If **p** then **r.**

If **q** then **s.**

Therefore, **r** or **s.**

Rhetorically, a dilemma is a choice between two options both of which have unappealing consequences. The pessimist philosopher Arthur Schopenhauer, for example, formulated what is sometimes called the "Hedgehog's dilemma," which we could paraphrase like this:

The closer two hedgehogs get, the more likely

> they are to poke each other with their spikes; but if they remain apart, they will be lonely. So it is with people: being close to someone inevitably creates conflicts and provocations and opens us to a lot of pain; but on the other hand, we're lonely when we stand apart.

In outline this argument might be put:

> Either we become **c**lose to others or we stand **a**part.
>
> If we become **c**lose to others, we **s**uffer conflict and pain.
>
> If we stand **a**part, we'll be **l**onely.
>
> Therefore, either we **s**uffer conflict and pain or we'll be **l**onely.

And in symbols:

> Either **c** or **a**.
>
> If **c** then **s**.
>
> If **a** then **l**.

Therefore, either **s** or **l.**

A further argument in dilemma form could conclude, even more simply, something like "Either way we'll be unhappy." I'll leave this one to you to write out formally.

Since this is such a jolly little conclusion, maybe I should add that hedgehogs are actually quite able to get close without poking each other. They can be together and comfortable too. Schopenhauer's second premise turns out to be false—at least for hedgehogs.

27 *Reductio ad absurdum*

One traditional deductive strategy deserves special mention even though, strictly speaking, it is only a version of *modus tollens*. This is the *reductio ad absurdum*, that is, a "reduction to absurdity." Arguments by *reductio* (or "indirect proof," as they're sometimes called) establish their conclusions by showing that assuming the opposite leads to absurdity: to a contradictory or silly result.

Nothing is left to do, the argument suggests, but to accept the conclusion.

To prove: **p.**

Assume the opposite: Not-**p.**

Argue that from the assumption we'd have to conclude: **q.**

Show that **q** *is false (contradictory, "absurd," morally or practically unacceptable ...).*

Conclude: **p** *must be true after all.*

Rule 12 discussed an argument for the existence of a Creator. Houses have creators, the argument goes, and the world is *like* a house—it too is ordered and beautiful. Thus, the analogy suggests, the world must have a Creator too. Rule 12 also cited David Hume's argument that the world is not relevantly similar enough to a house for this analogy to succeed. In Part V of his *Dialogues*, Hume also suggested a *reductio ad absurdum* of the

analogy.[8] Developed, it goes something like this:

> Suppose the world has a Creator like a house does. Now, when houses are not perfect, we know whom to blame: the carpenters and masons who created them. But the world is also not wholly perfect. Therefore, it would seem to follow that the Creator of the world is not perfect either. But you would consider this conclusion absurd. The only way to avoid the absurdity, however, is to reject the supposition that leads to it. Therefore, the world does not have a Creator in the way a house does.

Spelled out in *reductio* form, the argument is:

> *To prove:* The world does not have a Creator in the way a house does.
>
> *Assume the opposite:* The world does have a Creator in the way a house does.
>
> *Argue that from the assumption we'd have to conclude:* The Creator is imperfect (because the world is imperfect).

But: God cannot be imperfect.

Conclude: The world does not have a Creator in the way a house does.

Not everyone would find the idea of an imperfect God “absurd,” but Hume knew that the Christians with whom he was arguing would not accept it.

28 Deductive arguments in several steps

Many valid deductive arguments are combinations of the basic forms introduced in Rules 22–27. Here, for example, is Sherlock Holmes performing a simple deduction for Doctor Watson’s edification, meanwhile commenting on the relative roles of observation and deduction. Holmes has casually remarked that Watson visited a certain post office that morning, and furthermore that he sent off a telegram while there. “Right!” replies Watson, amazed, “Right on both points! But I confess that I don’t see

how you arrived at it." Holmes replies:

> "It is simplicity itself.... Observation tells me that you have a little reddish mold adhering to your instep. Just opposite the Wigmore Street Post Office they have taken up the pavement and thrown up some earth, which lies in such a way that it is difficult to avoid treading in it in entering. The earth is of this peculiar reddish tint which is found, as far as I know, nowhere else in the neighborhood. So much is observation. The rest is deduction."
>
> [Watson]: "How, then, did you deduce the telegram?"
>
> [Holmes]: "Why, of course I knew that you had not written a letter, since I sat opposite to you all morning. I see also in your open desk there that you have a sheet of stamps and a thick bundle of postcards. What could you go into the post office for, then, but to send a wire? Eliminate all other factors, and the one which remains must be the truth."[9]

Putting Holmes's deduction into explicit premises, we might have:

1. Watson has a little reddish mold on his boots.

2. If Watson has a little reddish mold on his boots, then he has been to the Wigmore Street Post Office this morning (because there and only there is reddish dirt of that sort thrown up, and in a way difficult to avoid stepping in).

3. If Watson has been to the Wigmore Street Post Office this morning, he either mailed a letter, bought stamps or cards, or sent a wire.

4. If Watson had mailed a letter, he would have written the letter this morning.

5. Watson wrote no letter this morning.

6. If Watson had bought stamps or cards, he would not already have a drawer full of stamps and cards.

7. Watson already has a drawer full of stamps and cards.

8. Therefore, Watson sent a wire at the Wigmore Street Post Office this morning.

We now need to break the argument down into a series of valid arguments in the simple forms presented in Rules 22–27. We might start with a *modus ponens:*

2. If Watson has a little reddish mold on his boots, then he has been to the Wigmore Street Post Office this morning.

1. Watson has a little reddish mold on his boots.

I. Therefore, Watson has been to Wigmore Street Post Office this morning.

(I will use I, II, etc. to stand for the conclusions of simple arguments, which then can be used as premises to draw further conclusions.)

Another *modus ponens* follows:

3. If Watson has been to the Wigmore Street Post Office this morning, he either mailed a letter, bought stamps or cards, or sent a wire.

I. Watson has been to Wigmore Street Post Office this morning.

II. Therefore, Watson either mailed a letter, bought stamps or cards, or sent a wire.

Two of these three possibilities now can be ruled out, both by *modus tollens:*

4. If Watson had gone to the post office to mail a letter, he would have written the letter this morning.

5. Watson wrote no letter this morning.

III. Therefore, Watson did not go to the post office to mail a letter.

and

6. If Watson had gone to the post office to buy stamps or cards, he would not already have a drawer full of stamps and cards.

7. Watson already has a drawer full of

stamps and cards.

IV. Therefore, Watson did not go to the post office to buy stamps or cards.

Finally we can put it all together:

II. Watson either mailed a letter, bought stamps or cards, or sent a wire at the Wigmore Street Post Office this morning.

III. Watson did not mail a letter.

IV. Watson did not buy stamps or cards.

8. Therefore, Watson sent a wire at the Wigmore Street Post Office this morning.

This last inference is an extended disjunctive syllogism: "Eliminate all other factors, and the one which remains must be the truth."

[8] David Hume, *Dialogues Concerning Natural Religion*, pp. 34–37.

[9] Sir Arthur Conan Doyle, "The Sign of Four," in *The Complete Sherlock Holmes*, pp. 91–92.

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